Channel Assignment with handover queueing in LEO Satellite Systems based on an "Earth-Fixed Cell" Coverage

L. Boukhatem[1,2], A.L. Beylot[1], D. Gaïti[2,3], and G. Pujolle[2]

[1]Laboratoire PRiSM, Université de Versailles - 45, Avenue des Etats-Unis 78035 Versailles – France

[2]Laboratoire LIP 6, Université de Paris 6 - 4, Place Jussieu 75252 Paris Cedex – France

[3]Laboratoire LM2S, Université de Technologie de Troyes - BP. 2060 10010 Troyes Cedex – France

Abstract

Low-Earth Orbit (LEO) satellite systems are intended to have an important part of the future generation of mobile telecommunication systems. They aim to provide different services to various populations of users. Each class of users requires a certain Quality of Service (QoS) and thus a given part of the shared channel resource. In this paper, LEO satellite systems based on an earth-fixed cell concept are considered, and different channel allocation strategies with handover queuing are discussed. Two channel allocation techniques have been investigated : fixed and dynamic channel allocation FCA and DCA. Moreover, in order to reduce the handover failure probability, we have assumed that handover attempts can be queued. An analytical model has been derived in the FCA case considering handover queuing and different categories of users. Implementation aspects for the DCA scheme have been discussed in comparison with FCA results.

Keywords – LEO, earth-fixed cells, handover, FCA, DCA.

1 Introduction

The increasing demand for mobile personal communications has involved many research and development efforts towards a new generation of mobile systems. Mobile Satellite Systems (MSSs) get an important part of interest in these studies. These systems will extend and complement the existing terrestrial cellular networks and provide global mobile telephony, data transmissions and mutimedia services for both mobile and fixed users especially those located in rural, sparsely populated and remote areas.

LEO satellites are placed on orbits with altitudes between 500 and 2000 km above the earth's surface. Compared to the geostationary orbit, the low orbital altitude means smaller end-to-end delays, lower power requirement for both satellites and handheld terminals, and a high degree of channel reusability (which increases the overall system capacity) [1, 2, 3, 4].

The footprint of each satellite can be divided into several cells, each one corresponding to a "spot-beam" of the satellite antenna. In LEO systems, two kinds of coverage concepts can be defined: *satellite-fixed cell* (SFC) and *earth-fixed cell* (EFC) coverage. The satellite-fixed cell concept corresponds to the case where beams remain constant relatively to the spacecraft and thus the corresponding cells on the ground move along with the satellite.

In SFC systems, as cells move relatively to the ground, the handover process is introduced by the satellite motion and not the motion of mobile users. Therefore, users will experience two kinds of handover: beam handover (from beam to beam) and satellite handover (from satellite to satellite). From a user point of view, it is important to notice that, unlike terrestrial systems, all users either fixed or mobile experience the handover procedure.

In earth-fixed cells systems, the earth's surface is divided into predetermined cells that have fixed boundaries, just like in terrestrial cellular networks. The relatively small fixed cells provide a means to contour service areas to country boundaries, and the type of services allowed within each cell is provided by an onboard database. In EFC systems, each satellite beam is assigned to a given ground cell for a fixed time period (beam steering phase). At the end of this time interval, all beams are reassigned to new adjacent cells (cell switching phase) [5].

Most of the under-developing non-GEO projects providing multimedia services have adopted the EFC concept as Teledesic, Skybridge, and M-star LEO systems. This paper mainly focuses on earth-fixed cell systems.

EFC systems are intended to provide different services for both fixed and mobile users. Our objectives in this paper is to study the performance, in terms of channel allocation, of a multimedia and broadband system which supports several classes of users. Moreover, we aim to study the joined effect of different channel allocation strategies and a queuing policy of handover attempts. We have considered fixed channel allocation (FCA) and dynamic channel allocation (DCA) strategies, and we have derived by simulation the performance of each technique. We have developed a mathematical model for the FCA scheme supporting the queuing strategy. A performance comparison of both FCA and DCA with handover queuing has been investigated by simulation under non-uniform traffic conditions and considering different classes of users.

The paper is organized as follows: Section 2 presents the handover procedure in EFC systems and describes the queuing policy of handover attempts. Section 3 gives some preliminary assumptions and presents a mathematical description of the model for the FCA technique considering handover queuing. Both FCA and DCA techniques are described in Section 4. Finally, section 5 deals with simulation results for FCA and DCA.

2 Handover in EFC systems

The great advantage of using earth-fixed cells is achieved when a mobile user experiments a beam or a satellite handover. With satellite-fixed cells, the handover procedure means that a new channel has to be allocated to the mobile user within the new beam or satellite. If no channel is available in the next serving beam or satellite, the handover procedure fails and the call is dropped.

In EFC systems, communication channels (frequencies and time slots) are permanently associated with each fixed cell and managed by the current serving satellite. As long as the terminal remains within the cell, it keeps the same channel during the call duration, whatever is the serving beam or satellite. Therefore, the EFC coverage offers significant advantages in terms of no handover failure probability for fixed users, and a low value for mobile ones.

Consequently, the handover failure probability, in an EFC context, depends on the number of mobile users which leave their cell during their communication's lifetime. Thus, this probability is a function of both users mobility and earth-fixed cell size. In under-developing EFC systems, cells sizes are quite small (53.3 km for Teledesic). Furthermore, systems designers are studying, for the future LEO satellite systems, a new generation of efficient satellites which use extremely narrow beam antennas able to cover very small areas on the earth's surface leading to an extremely efficient use of the spectrum. In such a context, the handover probability increases since the considered cell size is reduced. For our investigations, we consider small size cells systems.

2.1 Queuing handover attempts

From a user point of view, the most important performance criterion is the probability of forced call terminations. Therefore, to reduce this probability, a queuing procedure has been carried out. queuing of handover requests requires a given degree of overlap between the footprints of adjacent beams. The time spent by a mobile user to cross the overlap area defines the maximum waiting time for handover demands. This time depends on several parameters such as the user mobility and the overlap area extension crossed by the mobile user.

Concerning the access to the shared radio medium, we have considered that, in the uplink, an FDMA access is performed by the user terminals (as described in the Teledesic system[7]).

Let us assume that the entire bandwidth resource is divided into a fixed number of sub-channels (units), and each user with type i requires b_i units . We denote by $A(x)$ the number of available sub-channels for cell x at the call arrival instant in x. $A(x)$ is defined by the chosen channel allocation strategy (here FCA and DCA).

- Let us assume that a handover request of a mobile user with type i arrives in cell x, and requires b_i units of the shared bandwidth. If it results that $A(x) \geq b_i$, the user is accepted in cell x and the requested sub-channel(s) is(are) allocated to him. Otherwise, the handover attempt is queued in the handover queue (using a FIFO policy) waiting for an available sub-channel in cell x. If a sub-channel is released before the handover waiting time has expired, the call is served. Otherwise, the call is lost.

- Let us assume that a call termination of a user with type i occurs in cell x. This termination is due either to a handover or to the end of the call. In both cases, b_i units of the channel resource are released and can thus be allocated to a queued request.

3 Analytical approach

In this paper, the system is assumed to be composed of a set of adjacent square cells supporting a non-uniform traffic.

Moreover, we assume that the model supports different kinds of users. Fixed and mobile users are considered, and both types could also be divided into different kinds according to a given criterion (here, the bandwidth : the number of required sub-channels).

In this section, we develop an analytical model to derive the blocking probability for each class of users. We assume that the system supports k customer types and contains R cells, each one has a finite capacity of C sub-channels.

The model requires the following assumptions:

- New call arrivals for a type i user in cell j are assumed to be Poisson processes with a parameter $\lambda_{i,j,nc}$.

- Users of type i require b_i units of sub-channel resources.

- The sub-channel holding time in a cell by a type i user is exponentially distributed with a parameter $\mu_{i,h}$.

- The communication's lifetime of a type i user is exponentially distributed with a parameter $\mu_{i,c}$.

- The handover waiting time is limited and assumed to be exponentially distributed with a parameter $\mu_{i,w}$.

- $T_{jj'}$ denotes the probability for a given mobile user to go from cell j to cell j', and $N(j)$ is the set of neighbor cells of cell j.

Let us denote by $P_{i,j,b}$ the blocking probability of new call attempts of type i users in cell j, and $P_{i,j,h}$ the handover failure probability which corresponds to the fact that resources cannot be allocated to the user during his handover waiting period.

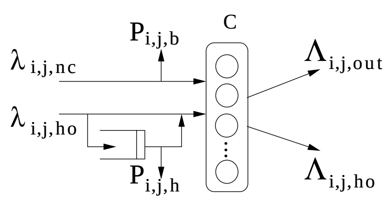

Figure 1: Cell model.

Figure 1 shows the different traffic components that require a sub-channel in a given cell j. We note that a given cell receives sub-channel requests due to new call attempts of different type i users and also the handover traffic coming from the adjacent cells. Let $\lambda_{i,j,ho}$ denotes the handover arrival rate in cell j for type i users.

The mean output rate can be expressed as follows:

$$\Lambda_{i,j} = \Lambda_{i,j,out} + \Lambda_{i,j,ho} \tag{1}$$

$$\Lambda_{i,j} = \lambda_{i,j,nc}(1 - P_{i,j,b}) + \lambda_{i,j,ho}(1 - P_{i,j,h}) \tag{2}$$

The output handover traffic rate of cell j is given by:

$$\Lambda_{i,j,ho} = \frac{\mu_{i,h}}{\mu_{i,h} + \mu_{i,c}}(\lambda_{i,j,nc}(1 - P_{i,j,b1}) + \lambda_{i,j,ho}(1 - P_{i,j,b2})). \tag{3}$$

We face here a fixed-point problem since the input handover traffic depends on the output one:

$$\lambda_{i,j,ho} = \sum_{j' \in N(j)} T_{j'j} \Lambda_{i,j',ho} \tag{4}$$

The problem can be solved using an iterative method through the following linear system [8]:

$$\begin{cases} \lambda^0_{i,j,ho} = \sum_{j' \in N(j)} \Lambda^0_{i,j',ho} T_{j'j} \\ \Lambda^0_{i,j,ho} = \frac{\mu_{i,h}}{\mu_{i,h}+\mu_{i,c}}(\lambda_{i,j,nc} + \lambda^0_{i,j,ho}) \end{cases} \tag{5}$$

In each step n of the iterative method, the value of $\lambda^n_{i,j,ho}$ is computed and compared to the one found in the previous step. The procedure is repeated until a convergence criterion ϵ is reached: $\| \lambda^{n+1}_{i,j,ho} - \lambda^n_{i,j,ho} \| < \epsilon$. The first value $\lambda^0_{i,j,ho}$ is computed disregarding the blocking probabilities as shown in system (5).

Once the handover arrival rate $\lambda^0_{i,j,ho}$ is derived, the blocking probability of each user class can be determined as follows.

The analytical structure of this problem is essentially the same as in a system where several types of customers share a finite group of servers, some of the customers may be queued but have a limited waiting time. In order to determine those parameters, we use a classical approximation, handover traffics are approximated by Poisson processes.

Two types of users are considered : M denotes mobile users and F corresponds to fixed users with higher rates supporting a wide range of fixed broadband services.

The analytical model is derived in the proposed study case but may be extended in a more general traffic case.

Let $N_{j,f}(t)$ and $N_{j,m}(t)$ denote respectively the number of fixed and mobile users in cell j at time t. Mobile users may either occupy sub-channels or wait for resources. Under the considered traffic conditions and the proposed approximations, the stochastic process $\{N_j(t) = (N_{j,f}(t), N_{j,m}(t)), t \in I\!R\}$ is a Markov process.

The set of allowable states, referred to as Δ, can be described as follows. Let $K_f = \lfloor \frac{C}{b_f} \rfloor$ denote the maximum number of fixed users that can be accepted. Thus, $\Delta = \{n = (n_f, n_m)/0 \leq n_f \leq K_f, n_m \in I\!N\}$.

An approximate aggregation method based on Courtois decomposition method [9] is used to solve this Markov chain and derive the performance criteria. It is described in Annex A.

At this step, the $P_{i,j,b}$ and $P_{i,j,h}$ values are determined using $\lambda^0_{i,j,ho}$. With these two values, $\Lambda^1_{i,j,ho}$ can be computed using system (5). The iterative procedure is repeated until the convergence criterion ϵ is reached.

4 Channel allocation techniques

4.1 Fixed channel allocation (FCA)

With fixed channel allocation, the full set of A available channels of the system is divided into K equal groups each composed of A/K channels. Regular groups of K cells (clusters) are formed such that the frequency reuse distance is maximized. However, K must be large enough to provide sufficient frequency reuse distance and guarantee the required minimum carrier to interference value $(C/I)_0$.

A set of A/K channels is permanently assigned to each cell. A new call can be served only if a free channel is available in the set of the cell.

For high network loads, fixed channel allocation is efficient, if the traffic is equally distributed among the cell. For a varying and non-uniform traffic, a complex planning is required to allocate more channels in the cells were a higher traffic is expected [10, 11].

4.2 Dynamic channel allocation (DCA)

In dynamic channel allocation, the assignment of channels to cells is based on the traffic demand in the cells. In other words, all channels are kept in a common pool and assignments are made in real time. Any channel can be temporarily allocated to any cell, provided that the constraint on the reuse distance is fulfilled (a given signal quality can be maintained). All DCA schemes evaluate the cost of using each available channel and choose the one which introduces the minimum cost.

Several DCA schemes were proposed. For our implementation we have chosen the algorithm described in [12]. The scheme uses a bookkeeping procedure that keeps track of the status and availability of channels in each cell. Further details on this algorithm are given in [12].

5 Simulation results

In this section, the performance of channel allocation techniques FCA and DCA have been derived by simulations. In particular, we have considered that the simulated cellular network is a grid of square shaped cells folded onto itself with six cells per side. The other system parameters values are shown in Table I. Moreover, we assumed an infinite queue capacity for handover requests.

Figure 2 compares analytical and simulation results in terms of new call blocking probability of fixed and mobile users (respectively $P_{f,b}$ and $P_{m,b}$) and handover blocking probability $P_{m,h}$. We can note that there is a good agreement between analytical predictions and simulation results. However, concerning $P_{m,h}$, there is a slight difference which is exclusively due to the pessimist approximation of handover arrivals to a Poisson traffic.

Figure 3 shows the different blocking probabilities as a function of the traffic load for FCA scheme. It plots the obtained results considering both cases with and without queuing (average queuing time of 2 seconds). We can easily note that the queuing strategy allows a significant reduction of $P_{m,h}$ without really affecting the values of $P_{f,b}$ and $P_{m,b}$. Furthermore, we can notice that the behavior of $P_{f,b}$ and $P_{m,b}$ are different; $P_{f,b}$ shows a higher blocking probability since fixed users require more sub-channel units than mobile users.

A performance comparison between FCA and DCA supporting the handover queuing is presented in Figure 4. The average waiting time parameter has been fixed to 2 and 3 seconds. The results show that DCA outperforms FCA in the traffic range under examination.

6 Conclusion

In this paper, a performance evaluation of fixed and dynamic channel allocation techniques with handover queuing has been addressed. The context of the study was a LEO satellite constellation system based on an earth-fixed cell concept. Two channel allocation schemes have been evaluated considering the case where handover requests are queued using a FIFO strategy. Furthermore, it has been assumed that the system supports different categories of fixed and mobile users. A mathematical model has been derived for the FCA strategy where both handover queuing and users diversity have been taken into account. Performance evaluations and comparisons have been carried out in terms of blocking probabilities of the different classes of users. In particular, we have proved by simulations that the DCA technique outperforms the FCA scheme under non uniform traffic conditions. Finally, we have shown that the queuing strategy enhances the performance of both the classical FCA and DCA schemes.

Table I: System parameters.

- two tiers of interfering cells (for FCA);
- average call duration: 3 min. for mobile users and 4 min. for fixed ones, average queuing time: 2 and 3 seconds;
- 20 sub-channels/cell are available with FCA;
- the proportions of users: 40% of type M (requiring 1 sub-channel) and 60% of type F (requiring 2 sub-channels);
- non-uniform traffic distribution.

References

[1] B. Gavish, "LEO/MEO systems - Global mobile communication systems", *Telecommunication Systems*, vol. 8, pp. 99-141, 1997.

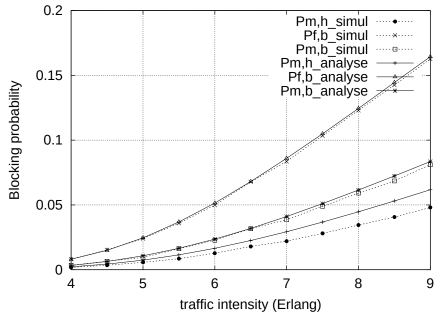

Figure 2: Simulation and analytical results (FCA)

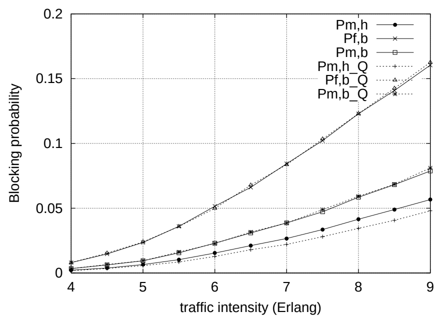

Figure 3: FCA, with and without queuing

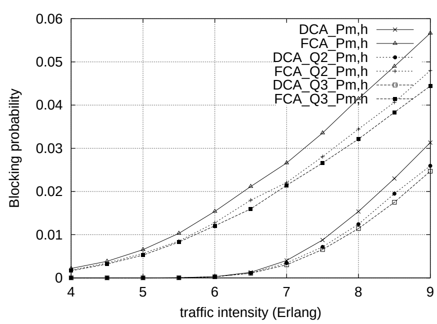

Figure 4: FCA versus DCA

[2] E. Lutz, "Issues in satellite personal communication systems", *Wireless Networks*, vol. 4, pp. 109-124, 1998.

[3] L.S. Golding, "Satellite communications systems move into the twenty-first century", *Wireless Networks*, vol. 4, pp. 101-107, 1998.

[4] F. Ananasso and F.D. Priscoli, "Satellite systems for personal communication networks", *Wireless Networks*, vol. 4, pp. 155-165, 1998.

[5] L. Boukhatem, A.L. Beylot, D. Gaïti, and G. Pujolle, "Performance Analysis of Dynamic and Fixed Channel Allocation Techniques in a LEO Constellation with an Earth-Fixed Cell System", *Globecom'00*, San Francisco, November 2000.

[6] J. Restrepo Mejia, "Comparative Analysis of Low Earth Orbit Satellite Constellations (Satellite-fixed and Earth-fixed Cells) for Fixed and Mobile Users", *Ph.D. Thesis*, ENST Telecom Paris, 1997.

[7] D.P. Patterson and M.A. Sturza, "Earth-Fixed Cell Beam Management For Satellite Communication System", *U.S patent*, no. 5.408.237, April 1995.

[8] S. Boumerdassi and A.L. Beylot, "Adaptive Channel Allocation for Wireless PCN", *ACM Journal on Special Topics in Mobile Networks and Applications, MONET*, Vol. 4, pp. 111-116, 1999.

[9] P.J Courtois, "Decomposability: queuing and computer systems applications", *Academic Press*, London, 1977.

[10] E. Del Re, R. Fantacci, and G. Giambene, "Efficient Dynamic Channel Allocation Techniques with Handover queuing for Mobile Satellite Networks", *IEEE Journal on Selected Areas in Communications*, vol. 13, no. 2, February 1995.

[11] E. Del Re, R. Fantacci, and G. Giambene, "Handover Queuing Strategies with Dynamic and Fixed Channel Allocation Techniques in Low Earth Orbit Mobile Satellite Systems", *IEEE Transactions on Communications*, vol. 47, no. 1, January 1999.

[12] D.D. Dimitrijevic and J. Vucetic, "Design and Performance Analysis of the Algorithms for Channel Allocation in Cellular Networks", *IEEE Transactions on Vehicular Technology*, Vol. 42, no. 4, pp. 526-534, November 1993.

Annex A

In order to simplify the notations, the dependence on the cell j has been omitted. Index f and m refer respectively to fixed and mobile users. The structure of the graph corresponding to the Markov process $N(t)$ is too complex to derive an exact solution. Consequently, we propose an approximate solution based on the following remarks.

- In the case when, $1 \leq n_f \leq K_f$ and $n_m > 0$, state $n = (n_f, n_m)$ is connected to states (n_f+1, n_m), (n_f-1, n_m) (n_f, n_m+1), (n_f, n_m-1).
- In the case when at least one handover is waiting, only the last three ones are reachable.
- $\mu_m = \mu_{m,c} + \mu_{m,h} \ll \mu_{m,w}$ and $\mu_{f,c} \ll \mu_{m,w}$.

We consequently suggest the following decomposition. Let Γ denote the set of states for which the handover queue is empty: $\Gamma = \{n = (n_f, n_m) / n_f b_f + n_m b_m \leq C\}$.

Let Ω_k denote the set of states for which there are k fixed users in the considered cell and for which the handover queue is not empty.

$$\Delta = \Gamma \cup \Omega_1 \cup \ldots \cup \Omega_{K_f}.$$

The method consists on decomposing the original infinitesimal generator Q into blocks. Each block corresponds to one of the previous set of states. The aggregation technique leads to the following two steps.

Decomposition Phase

In this first step, we solve the unnormalized systems

$$\pi_\delta Q^*_\delta = 0,$$

where π_δ denotes the vector of steady state probabilities of the different states of aggregate δ and Q^*_δ is the approximate infinitesimal generator of aggregate δ defined as follows:

$$\begin{cases} q^*_{ij} = q_{ij}, (i,j) \in \delta^2, i \neq j \\ q^*_{ii} = -\sum_{j\in\delta, i\neq j} q_{ij} \end{cases}$$

The solution of those systems leads to the determination of the steady state probabilities of the different states as a function of a constant which may be the steady state probability of beeing in aggregate δ. It can easily be shown that aggregates Ω_k subchains are of birth-death process type. The solution of the previous systems leads to

$$\pi_{k,M_k+j} = \prod_{r=2}^{j} \frac{\lambda_{m,ho}}{M_k\mu_m + r\mu_{m,w}} \pi_{k,M_k+1},$$

where $\pi_{k,l}$ is the steady state probability of state (k,l) $M(k) = \lfloor \frac{C-k.b_f}{b_m} \rfloor$.

Let $\Pi(\Omega_k)$ denote the steady state probability of beeing in one of the states of aggregates Ω_k, it can be shown that

$$\begin{aligned} \Pi(\Omega_k) &= \sum_{j=1}^{+\infty} \pi_{k,M_k+j} \\ &= \pi_{k,M_k+1}\{1 + \sum_{j=2}^{+\infty}\prod_{r=2}^{j} \frac{\lambda_{m,ho}}{M_k\mu_m + r\mu_{m,w}}\} \end{aligned}$$

which may be approximated if $M_k\mu_m \ll \mu_{m,w}$ by

$$\Pi(\Omega_k) \simeq \frac{\pi_{k,M_k+1}}{\rho_{m,h}}(e^{\rho_{m,h}} - 1), \quad \text{with } \rho_{m,h} = \frac{\lambda_{m,ho}}{\mu_{m,w}}.$$

For the subchain corresponding to aggregate Γ, one can easily find that:

$$\pi_{k,j} = \pi_{0,0}\frac{\rho_f^k}{k!}\frac{\rho_m^j}{j!},$$

where $\rho_m = \frac{\lambda_{m,nc}+\lambda_{m,ho}}{\mu_m}$ and $\rho_f = \frac{\lambda_{f,nc}}{\mu_f}$.

Consequently, the steady state probability of beeing in aggregate Γ is:

$$\Pi(\Gamma) = \pi_{0,0}\sum_{k=0}^{K_f}\sum_{j=0}^{M_k}\frac{\rho_f^k}{k!}\frac{\rho_m^j}{j!}.$$

Aggregation Phase

In the second step, we shall find relations between the different aggregates.

Let us note: $\Theta_\Gamma = \frac{\Pi(\Gamma)}{\pi_{0,0}}$ and $\Theta_{\Omega_k} = \frac{\Pi(\Omega_k)}{\pi_{k,M_k+1}}$.

Using the Chapmann Kolmogorov equations, we can derive:

$$\begin{aligned} \lambda_{m,ho}\pi_{K_f,M_{K_f}} &= (M_{K_f}\mu_m + \mu_{m,h})\pi_{K_f,M_{K_f}+1} \\ &+ K_f\mu_f \sum_{j=M_{K_f}+1}^{+\infty} \pi_{K_f,M_{K_f}+j} \end{aligned}$$

which allows to express $\Pi(\Omega_{K_f})$ as a function of $\Pi(\Gamma)$:

$$\frac{\lambda_{m,ho}}{\Theta_\Gamma}\frac{\rho_f^{K_f}}{K_f!}\frac{\rho_m^{M_{K_f}}}{M_{K_f}!}\Pi(\Gamma) = \{\frac{M_{K_f}\mu_m}{\Theta_{\Omega_{K_f}}} + K_f\mu_f\}\Pi(\Omega_{K_f}).$$

Using an iterative method, we can find:

$$\begin{aligned} &\lambda_{m,ho}\pi_{(k,M_k)} + (k+1)\mu_f \sum_{j=M_k+1}^{+\infty} \pi_{k+1,M_k+j} \\ &= (M_k\mu_m + \mu_{m,h})\pi_{k,M_k+1} + k\mu_f \sum_{j=M_k+1}^{+\infty} \pi_{k,M_k+j} \end{aligned}$$

which leads to an expression of $\Pi(\Omega_k)$ as a function of $\Pi(\Gamma)$. Using the equation of normalization:

$$\Pi(\Gamma) + \sum_{k=0}^{K_f}\Pi(\Omega_k) = 1,$$

the steady state probabilites and the performance criteria can consequently be derived.

Performance criteria determination

We are supposed to compute the new call blocking and the handover failure probabilities. The probability for a new call to be accepted is the probability that, when a new call arrives, the available bandwidth is greater than the required bandwidth. Since, new call arrivals are assumed to be Poisson, PASTA property leads to:

$$\begin{cases} P_{f,b} = 1 - \sum_{k=0}^{K_f}\sum_{j=0}^{M_k}\pi_{k,j} \\ P_{m,b} = 1 - \sum_{j=0}^{K_m}\sum_{k=0}^{F_j}\pi_{k,j} \end{cases}$$

where F_k and K_m are defined in the same way as M_k and K_f, $K_m = \lfloor \frac{C}{b_m} \rfloor$, $F_k = \lfloor \frac{C-k.b_f}{b_m} \rfloor$.

The handover failure probability depends on the handover flow accepted in the different states: when handover calls are accepted, the accepted flow is $\lambda_{m,ho}$. When handover traffic is queued, this rate will depend on the departure rates of calls. When a mobile user will leave a cell or finish his call, the sub-channels will be allocated to the first handover which is queued. When a fixed user will finish his call, several handover calls may be dequeued.

We obtain the accepted handover rate $\lambda_{m,a}$:

$$\begin{aligned} \lambda_{m,a} = &\lambda_{m,ho}\sum_{k=0}^{K_f}\sum_{j=0}^{M_k-1}\pi_{k,j} \\ &+ \sum_{k=0}^{K_f}\sum_{j=M_k+1}^{+\infty}\pi_{k,j}(M_k\mu_M + k\mu_f a_{k,j}) \end{aligned}$$

where $a_{k,j} = Min\{j - M_k, \lfloor \frac{b_f + (C-b_f k - b_m j)}{b_2} \rfloor\}$.

We finally obtain the handover failure probability:

$$P_{m,h} = 1 - \frac{\lambda_{m,a}}{\lambda_{m,ho}}.$$